Observability Analysis and Optimization for Angles-Only Navigation of Distributed Space Systems

Justin Kruger[*]**and Simone D'Amico**[†]

keywords: angles-only navigation, observability analysis, design optimization, , ,

Angles-only methods, in which observer spacecraft obtain bearing angles to target space objects using on-board vision-based sensors, are compelling for swarm and constellation navigation. Navigation is both distributed and self-contained with resultant advantages in autonomy, responsiveness and scalability; and cameras are ubiquitous, accurate, miniaturized sensors with passive measurement capability. Such aspects have spurred development of new angles-only architectures and mission proposals.[3,4] However, bearing angles are also characterized by weak observability. System properties such as orbit geometry, sensor quality and measurement availability can affect state observability significantly and unpredictably.[4,5] Despite this complexity, comparatively little attention has been paid to the critical problem of how angles-only systems may be practically designed to achieve the required navigation performance for a scenario. In response, this paper constructs a new, unified angles-only observability analysis and design framework from three interrelated components: **analytic observability** analysis, **numeric observability** analysis, and **global optimization**. Analysis methods are modified for a more general and flexible treatment of observability, thus enabling designers to 1) analytically determine whether the system is observable, 2) numerically estimate expected navigation performance, and 3) simply and intelligently optimize the system to meet navigation requirements.

Analytic observability is investigated by developing a system graph topology representation. Observers and targets are graph nodes and directed graph edges display measurement flow from target to observer. In contrast to previous research,[1] global instantaneous measurement availability is not assumed for a more realistic treatment of distributed systems. Weighted and self-loop edges are introduced to characterize an observer receiving 1) multiple measurements of a target from remote observers, and 2) measurements of itself from remote observers, both of which improve observability. A node's subgraph contains itself, its targets, and remote observers it receives measurements from. Resultant conditions for subgraph observability stem from the Lie-derivative criterion,[1] where observability matrix $\mathcal{O}_q$ is full rank if the system is q'th-order locally weakly observable. For state $\boldsymbol{x}$, measurement model $\boldsymbol{h}(\boldsymbol{x})$, and dynamics model $\boldsymbol{f}(\boldsymbol{x})$, the matrix $\mathcal{O}_q$ is

$$\mathcal{O}_q = \frac{\partial}{\partial \boldsymbol{x}} \begin{bmatrix} \boldsymbol{h}(\boldsymbol{x}) \\ \frac{\partial \boldsymbol{h}(\boldsymbol{x})}{\partial \boldsymbol{x}} \boldsymbol{f}(\boldsymbol{x}) \\ \frac{\partial^2 \boldsymbol{h}(\boldsymbol{x})}{\partial \boldsymbol{x}^2} \boldsymbol{f}(\boldsymbol{x})^2 \\ \vdots \\ \frac{\partial^q \boldsymbol{h}(\boldsymbol{x})}{\partial \boldsymbol{x}^q} \boldsymbol{f}(\boldsymbol{x})^q \end{bmatrix} \qquad [1]$$

An observer is a 'beacon' and attains absolute orbit observability for itself and its targets if $\mathcal{O}_q$ for its subgraph is full rank. Topologically, the observer must fulfil at least one of the following conditions:

1. It is the sink of an edge with weight $w \geq 2$ and the measurement source is a target.
2. It is the sink of an edge with weight $w \geq 2$ and the measurement source is a beacon.
3. It possesses a self-loop edge and the measurement source is a beacon.

If every node is itself a beacon or the target of a beacon, the system is observable by definition. In Figure 1, Observer 3 gains beacon status via Conditions 1-3.

Numeric observability for observer subgraphs is assessed by computing a lower bound for estimated state covariance $\boldsymbol{P}_{\text{est}}$ using the observer measurement sensitivity matrix $\boldsymbol{Y}_{\text{est}}$ and measurement noise matrix $\boldsymbol{R}$.[2] Noise contributions include sensor noise $\boldsymbol{R}_{\text{meas}}$, uncertainty in a-priori information $\boldsymbol{R}_{\text{prior}}$, and dynamics noise $\boldsymbol{R}_{\text{dyn}}$. Past work has assumed $\boldsymbol{R}_{\text{dyn}} = \boldsymbol{0}$, whereas this analysis introduces time-varying physically correlated dynamics noise to account for mismodeled perturbations. $\boldsymbol{P}_{\text{est}}$ is computed from

$$(\boldsymbol{R}_{\text{meas}} + \boldsymbol{R}_{\text{prior}} + \boldsymbol{R}_{\text{dyn}}) = \boldsymbol{Y}_{\text{est}} \boldsymbol{P}_{\text{est}} \boldsymbol{Y}_{\text{est}}^\top \qquad [2]$$

[*]Stanford University, United States of America, jjkruger@stanford.edu

[†]Stanford University, United States of America, damicos@stanford.edu

 REFERENCES

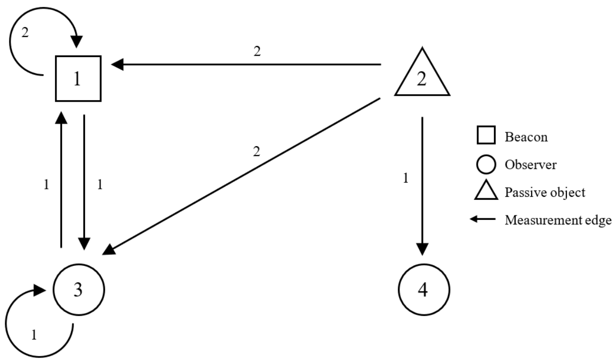

Fig. 1: Topology for an example four-spacecraft system. Edges are labelled with their weight.

Quantitative navigation requirements can then be directly related to state components in $\boldsymbol{P}_{\text{est}}$ with inclusion of realistic system-level errors and uncertainties. New observability assessments are performed for auxiliary states including inter-satellite clock offsets, ballistic coefficients and sensor biases. Combined orbit and auxiliary states are observable when there exists sufficient relative motion between objects. To prove the analytic observability conditions, all two-, three- and four-object subgraphs are enumerated and $\boldsymbol{P}_{\text{est}}$ is computed. Systems which fulfil at least one topological condition possess a maximum position uncertainty of <0.05% of the orbit radius. Systems which do not possess a minimum position uncertainty of >50% of the orbit radius. The conditions are therefore considered sufficient for observability.

Global optimization is performed by fusing the analytic and numeric methods into a unified tool such that angles-only systems can be automatically optimized to fulfil mission requirements. The objective function is quadratic as per $J_\sigma = \boldsymbol{\sigma}_{\max}^\top \boldsymbol{Q}_\sigma \boldsymbol{\sigma}_{\max}$, where $\boldsymbol{\sigma}_{\max}$ is worst-case state uncertainty across all system members and $\boldsymbol{Q}_\sigma$ is a weighting matrix. Optional cost terms account for system size; deviation from nominal orbits; sensor quality; communication range; and visual coverage, versus corresponding performance improvement. Optional penalty terms ensure maximum allowable state uncertainty is met and redundancy in case of observer failure. Gradient descent minimizes the defined cost. Analytic methods provide an initial indication of behavior and graph edges are dependent on target visibility as per sensor attitude, field of view (FOV), target visual magnitude, and eclipse periods. Numeric methods provide the final output and include spherical harmonic gravity, atmospheric drag, solar radiation pressure (SRP) and third-body gravity perturbations, with default bearing angle measurement noise of $20''$ and orbit-specific dynamics noise.

The automatic optimization is applied to two case studies. First is a distributed science swarm in low Mars orbit desiring position uncertainty of <500m, clock uncertainty of <0.1s, and adherence to a nominal in-train formation. The swarm is initialized as three spacecraft. Preliminary tests meet requirements by expanding to six spacecraft with higher-quality sensing (i.e. halved measurement noise) and relative motion of ∼500m in magnitude. Second is an SDA constellation in frozen lunar orbit, desiring position uncertainty of <100m with estimation of SRP ballistic coefficients and redundancy in case of single-observer failure. The constellation is initialized as four observers in equally-spaced orbital planes. Preliminary tests meet requirements by expanding to four observers per plane, with a minimum 30°FOV to maintain target visibility without active pointing.

In conclusion, achieving sufficient observability for angles-only navigation presents a complex design problem. This work extends analytic and numeric analysis methods in combination with a flexible optimization framework to propose a powerful tool for distributed system design under realistic operational constraints.

References

[1] Yunpeng Hu, Inna Sharf, and Lei Chen. Distributed orbit determination and observability analysis for satellite constellations with angles-only measurements. *Automatica*, 129, 2021.

[2] A. W. Koenig and Simone D'Amico. Observability-Aware Numerical Algorithm for Angles-Only Initial Relative Orbit Determination. In *2020 AAS/AIAA Astrodynamics Specialist Conference*, Lake Tahoe, California, 2020.

[3] Adam W. Koenig, Justin Kruger, and Simone D'Amico. ARTMS: Enabling Autonomous Distributed Angles-Only Orbit Estimation for Spacecraft Swarms. In *American Control Conference*, New Orleans, Louisiana, 2021.

[4] Justin Kruger, Katherine Wallace, Adam W. Koenig, and Simone D'Amico. Autonomous Angles-Only Navigation for Spacecraft Swarms around Planetary Bodies. In *2021 IEEE Aerospace Conference*, Big Sky, Montana, 2021.

[5] David C. Woffinden and David K. Geller. Observability Criteria for Angles-Only Navigation. *IEEE Transactions on Aerospace and Electronic Systems*, 45(3):1194–1208, 2009.